

# STELLAR FEEDBACK IN MOLECULAR CLOUDS AND ITS INFLUENCE ON THE MASS FUNCTION OF YOUNG STAR CLUSTERS

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## ABSTRACT

We investigate how the removal of interstellar material by stellar feedback limits the efficiency of star formation in molecular clouds and how this determines the shape of the mass function of young star clusters. In particular, we derive relations between the power-law exponents of the mass functions of the clouds and clusters in the limiting regimes in which the feedback is energy-driven and momentum-driven, corresponding to minimum and maximum radiative losses, and likely to bracket all realistic cases. We find good agreement between the predicted and observed exponents, especially for momentum-driven feedback, provided the protoclusters have roughly constant mean surface density, as indicated by observations of the star-forming clumps within molecular clouds. We also consider a variety of specific feedback mechanisms, concluding that H II regions inflated by radiation pressure predominate in massive protoclusters, a momentum-limited process when photons can escape after only a few interactions with dust grains. We show in this case that the star formation efficiency depends on the masses and sizes of the protoclusters only through their mean surface density, thus ensuring consistency between the observed exponents of the mass functions of the clouds and clusters. Our numerical estimate of this efficiency is also consistent with observations.

*Subject headings:* galaxies: star clusters — HII regions — ISM: bubbles — radiative transfer — stars: formation — stars: winds, outflows

## 1. INTRODUCTION

Most stars form in protoclusters in dense molecular clumps (Lada & Lada 2003; McKee & Ostriker 2007). The energy and momentum injected by young stars then removes the remaining interstellar material (ISM), thus ending further star formation and reducing the gravitational binding energy of the protoclusters. This feedback limits the efficiency of star formation—the ratio of final stellar mass to initial interstellar mass—to only 20–30%, and leaves many protoclusters unbound, with their constituent stars free to disperse. Even those protoclusters that survive will lose some stars by ISM removal and subsequent processes.

Two of the best probes of these formation and disruption processes are the mass functions of molecular clouds and young star clusters, defined as the number of objects per unit mass,  $\psi(M) \equiv dN/dM$ . For molecular clouds, the best-studied galaxies are the Milky Way and the Large Magellanic Cloud (LMC), while for star clusters, they are the Antennae and the LMC. In these and other cases, the observed mass functions can be represented by power laws,  $\psi(M) \propto M^\beta$ , from  $10^4 M_\odot$  or below to  $10^6 M_\odot$  or above. Giant molecular clouds (GMCs) identified in CO surveys have  $\beta \approx -1.7$  (Rosolowsky 2005; Blitz et al. 2007; Fukui et al. 2008). This exponent is also found for massive self-gravitating clumps within GMCs, the formation sites of star clusters, whether they are identified by CO emission (Bertoldi & McKee 1992) or higher-density tracers such as C<sup>18</sup>O, <sup>13</sup>CO, and thermal

dust emission (Reid & Wilson 2006; Muñoz et al. 2007; Wong et al. 2008). Young star clusters have  $\beta \approx -2.0$  (Elmegreen & Efremov 1997; McKee & Williams 1997; Zhang & Fall 1999; Dowell et al. 2008; Fall et al. 2009; Chandar et al. 2010). The similar exponents for clouds and clusters indicate that the efficiency of star formation and probability of disruption are at most weak functions of mass. This conclusion is reinforced by the fact that  $\beta$  is the same for  $10^7 - 10^8$  yr-old clusters as it is for  $10^6 - 10^7$  yr-old clusters (Zhang & Fall 1999; Fall et al. 2009; Chandar et al. 2010).

These empirical results may at first seem puzzling. Low-mass protoclusters have lower binding energy per unit mass and should therefore be easier to disrupt than high-mass protoclusters. Indeed, several authors have proposed that feedback would cause a bend in the mass function of young clusters at  $M \sim 10^5 M_\odot$ , motivated in part by the well-known turnover in the mass function of old globular clusters (Kroupa & Boily 2002; Baumgardt et al. 2008; Parmentier et al. 2008). For young clusters, such a feature is not observed (as noted above), while for globular clusters, it arises from almost any initial conditions as a consequence of stellar escape driven by two-body relaxation over  $\sim 10^{10}$  yr (Fall & Zhang 2001; McLaughlin & Fall 2008, and references therein). Nevertheless, we are left with an important question: What are the physical reasons for the observed similarity of the mass functions of molecular clouds and young star clusters?

The goal of this Letter is to answer this question. In Section 2, we derive some general relations between the mass functions of clouds and clusters. In Section 3, we review a variety of specific feedback processes and estimate the star formation efficiency for radiation pressure, the dominant process in massive, compact protoclusters. We summarize in Section 4.

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## 2. MASS FUNCTIONS

The radiative losses inside protoclusters determine how much of the energy input by stellar feedback is available for ISM removal. This in turn depends on the cloud structure and the specific feedback mechanisms involved, but two limiting regimes bracket all realistic situations: energy-driven, with no radiative losses, and momentum-driven, with maximum radiative losses. We estimate the mass of stars  $M_*$  and the corresponding efficiency of star formation,  $\mathcal{E} = M_*/M$ , needed to remove the ISM from protoclusters in these regimes as follows. We characterize a protocluster by its mass  $M$ , half-mass radius  $R_h$ , mean surface density  $\Sigma$ , velocity dispersion  $V_m$  (including the orbital motions of the stars and the turbulent and thermal motions of the interstellar particles), RMS escape velocity  $V_e$ , and crossing time  $\tau_c$ . For simplicity, we neglect rotation, magnetic support, and external pressure (but see Section 3). Then the properties of a protocluster are related by  $V_m^2 = 0.4GM/R_h$ ,  $V_e = 2V_m$ ,  $\tau_c = R_h/V_m$  (Spitzer 1987) and  $\Sigma \approx (M/2)/(\pi R_h^2)$ . We also assume that the sizes and masses of protoclusters are correlated, with a power-law trend,  $R_h \propto M^\alpha$ .

In Figure 1, we plot  $\Sigma$  and  $R_h$  against  $M$  for star-forming molecular clumps in the Milky Way, based on measurements of CS,  $C^{17}O$ , and 1.2 mm dust emission in three independent surveys (Shirley et al. 2003; Faúndez et al. 2004; Fontani et al. 2005). These clumps were selected for their star-formation activity (water masers, IRAS colors), not their surface density. Evidently, there is a strong correlation between  $R_h$  and  $M$ , and almost none between  $\Sigma$  and  $M$ , corresponding to  $\alpha \approx 1/2$ . The typical surface density is close to the value  $\Sigma \sim 1 \text{ g cm}^{-2}$  expected from theory (McKee & Tan 2003; Krumholz et al. 2007; Krumholz & McKee 2008).<sup>4</sup> We assume that the Milky Way relations also hold in other galaxies and extend up to  $\sim 10^6 M_\odot$ , although it is conceivable that they break down above  $\sim 10^5 M_\odot$ . Indeed, Baumgardt et al. (2008) and Parmentier et al. (2008) assume that  $R_h$  is not correlated with  $M$  (corresponding to  $\alpha = 0$ ), based on observations of gas-free clusters (e.g. Murray 2009). However, since ISM removal necessarily occurs during the earlier, gas-dominated phase,  $\alpha \approx 1/2$  seems more appropriate in the present context. As we show here,  $\alpha \approx 1/2$  is also needed to reconcile the observed mass functions of molecular clouds and star clusters.

The rates of energy and momentum input are proportional to the stellar mass<sup>5</sup>:  $\dot{E} \propto \mathcal{E}M$  and  $\dot{P} \propto \mathcal{E}M$ . We assume that the timescale for ISM removal is a few crossing times:  $\Delta t \sim (1-10) \times \tau_c$  (Elmegreen 2000, 2007; Hartmann et al. 2001; Tan et al. 2006; Krumholz & Tan 2007). Thus, the total energy and momentum input are  $E \approx \dot{E}\Delta t \propto \mathcal{E}MR_h/V_m$  and  $P \approx \dot{P}\Delta t \propto \mathcal{E}MR_h/V_m$ . These reach the critical values needed to remove the ISM,  $E_{\text{crit}} = \frac{1}{2}MV_e^2$  and  $P_{\text{crit}} = MV_e$ , for

$$\mathcal{E} \propto V_e^3/R_h \propto M^{(3-5\alpha)/2} \quad (\text{energy-driven}), \quad (1a)$$

$$\mathcal{E} \propto V_e^2/R_h \propto M^{1-2\alpha} \quad (\text{momentum-driven}). \quad (1b)$$

<sup>4</sup> For reference, the Larson (1981) relation for CO-selected clouds corresponds to a much lower surface density,  $\Sigma \sim 0.02 \text{ g cm}^{-2}$ .

<sup>5</sup> This is a good approximation for all feedback mechanisms except protostellar outflows, which inject energy and momentum in proportion to the star formation rate. Outflows, however, are non-dominant in massive protoclusters; see Table 1.

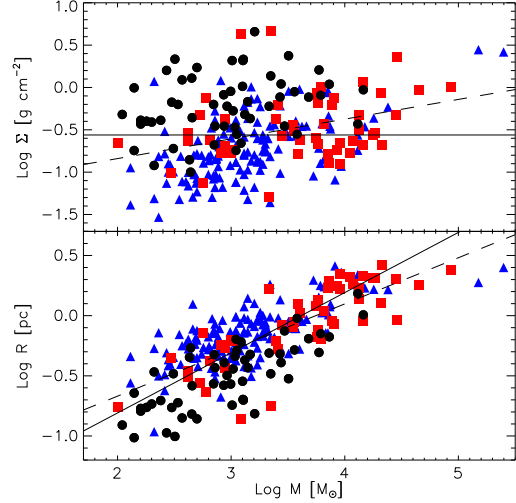


FIG. 1.— Surface density  $\Sigma$  and radius  $R$  plotted against mass  $M$  for star-forming molecular clumps from measurements by Shirley et al. (2003; circles, CS emission), Faúndez et al. (2004; triangles, dust emission), and Fontani et al. (2005; squares,  $C^{17}O$  and dust emission). We exclude clouds with  $M < 100 M_\odot$ , since they cannot form clusters. The lines are least-squares regressions ( $\log R$  against  $\log M$ ) with  $\alpha = 0.5$  fixed (solid) and  $\alpha = 0.38 \pm 0.023$  (dashed). The true uncertainty on  $\alpha$  is undoubtedly larger than the quoted one-sigma error.

For  $\alpha = 1/2$ , the efficiency has little or no dependence on mass:  $\mathcal{E} \propto M^{1/4}$  in the energy-driven regime,  $\mathcal{E} = \text{constant}$  in the momentum-driven regime. For  $\alpha = 0$ , the variation is much stronger:  $\mathcal{E} \propto M^{3/2}$  and  $\mathcal{E} \propto M$ , respectively. These relations are valid for  $\mathcal{E} \lesssim 0.5$ .

Any dependence of  $\mathcal{E}$  on  $M$  will cause the mass functions of star clusters  $\psi_*(M_*)$  and molecular clouds  $\psi(M)$  to have different shapes. For the moment, we confine our attention to clusters young enough to be easily recognizable even if they are unbound and dispersing. This limit is  $\sim 10^7$  yr for extragalactic clusters such as those in the Antennae (Fall et al. 2005). In this case, the mass functions of the clusters and clouds are related by  $\psi_*(M_*)dM_* \propto \psi(M)dM$  (with a coefficient greater than unity if several clusters form within each cloud). For  $\psi(M) \propto M^\beta$  and  $\mathcal{E} \propto M^\gamma$ , we have  $\psi_*(M_*) \propto M_*^{\beta_*}$  with  $\beta_* = (\beta - \gamma)/(1 + \gamma)$ . Equations (1a) and (1b) then imply

$$\beta_* = \frac{2\beta + 5\alpha - 3}{5(1 - \alpha)} \quad (\text{energy-driven}), \quad (2a)$$

$$\beta_* = \frac{\beta + 2\alpha - 1}{2(1 - \alpha)} \quad (\text{momentum-driven}). \quad (2b)$$

These expressions give  $\beta_* = \beta$  for  $\alpha = 3/5$  and  $1/2$ , respectively. Thus, the similarity of the mass functions of clusters and clouds ( $\beta_* \approx \beta$ ) requires that the latter have approximately constant mean surface density ( $0.5 \lesssim \alpha \lesssim 0.6$ ), no matter what type of feedback is involved.

Before proceeding, we make a small correction. For clouds, the observed mass function  $\psi_o(M)$  represents the true mass function at formation  $\psi(M)$  (i.e., the birthrate) weighted by the lifetime:  $\psi_o(M) \propto \psi(M)\tau_l(M)$ . We assume, as before, that lifetime is proportional to crossing time:  $\tau_l \propto \tau_c \propto M^{(3\alpha-1)/2}$ . Then

the exponents of the true and observed mass functions are related by  $\beta = \beta_o - (3\alpha - 1)/2$ . Inserting this into Equations (2a) and (2b), we obtain

$$\beta_* = \frac{2(\beta_o + \alpha - 1)}{5(1 - \alpha)} \quad (\text{energy - driven}), \quad (3a)$$

$$\beta_* = \frac{2\beta_o + \alpha - 1}{4(1 - \alpha)} \quad (\text{momentum - driven}). \quad (3b)$$

We now evaluate Equations (3a) and (3b) with  $\beta_o = -1.7$ , the observed exponent of the mass function of molecular clouds (Rosolowsky 2005; Reid & Wilson 2006; Muñoz et al. 2007; Wong et al. 2008; Fukui et al. 2008). For constant mean surface density ( $\alpha = 1/2$ ), we find  $\beta_* = -1.8$  in the energy-driven regime and  $\beta_* = -2.0$  in the momentum-driven regime. These predictions agree nicely with the observed exponents of the mass functions of young star clusters,  $\beta_* \approx -2.0$  (with typical uncertainty  $\Delta\beta_* \approx 0.2$ ). Our model is clearly idealized, but the scalings, and thus the agreement between the predicted and observed  $\beta_*$ , should be robust. For constant size ( $\alpha = 0$ ), however, we find  $\beta_* = -1.1$  in both the energy-driven and momentum-driven regimes, in definite conflict with observations.

The mass function of star clusters older than  $\sim 10^7$  yr depends on the proportion that remain gravitationally bound. This in turn depends on the efficiency of star formation  $\mathcal{E}$  and the timescale for ISM removal  $\Delta t$  relative to the crossing time  $\tau_c$ . Both analytical arguments and  $N$ -body simulations indicate that young clusters lose most of their stars for  $\mathcal{E} \lesssim 0.3$  and  $\Delta t \ll \tau_c$  but retain most of them for  $\mathcal{E} \gtrsim 0.5$  or  $\Delta t \gg \tau_c$  (Hills 1980; Kroupa et al. 2001; Kroupa & Boily 2002; Baumgardt & Kroupa 2007). Thus, as long as  $\mathcal{E}$  and  $\Delta t/\tau_c$  are, on average, independent of  $M$ , as they are for protoclusters with constant mean surface density ( $\alpha = 1/2$ ) and momentum-driven feedback, ISM removal will not alter the shape of the mass function (although its amplitude will decline). This is consistent with the observed exponents  $\beta_* \approx -2.0$  for clusters both younger and older than  $10^7$  yr in the Antennae and LMC (Zhang & Fall 1999; Fall et al. 2009; Chandar et al. 2010).

In all other cases,  $\mathcal{E}$  increases with  $M$ , and a higher proportion of low-mass clusters is disrupted, causing a flattening or a bend at  $\mathcal{E} \approx 0.3 - 0.5$  in the mass function. The exact shape depends on  $\Delta t/\tau_c$ , clumpiness within protoclusters, and other uncertain factors. If the efficiency has a weak dependence on mass, as it does for constant mean surface density ( $\alpha = 1/2$ ) and energy-driven feedback ( $\mathcal{E} \propto M^{1/4}$ ), the predicted  $\beta_*$  might be marginally consistent with observations over a limited range of masses ( $10^4 M_\odot \lesssim M \lesssim 10^6 M_\odot$ ). However, for constant size ( $\alpha = 0$ ), the variations are so strong ( $\mathcal{E} \propto M^{3/2}$  and  $\mathcal{E} \propto M$ ) that we expect major differences between the mass functions of clusters younger and older than  $10^7$  yr, in clear contradiction with observations.

Our simple analytical model agrees, at least qualitatively, with the numerical calculations by Baumgardt et al. (2008) and Parmentier et al. (2008). They present results for energy-driven feedback by supernovae in protoclusters with uncorrelated sizes and masses. In some cases, they find a bend in the mass function of young clusters at  $M \sim 10^5 M_\odot$ , while in others, they find a flattened power law with  $\beta_* \approx -1$  (see Figure 4 of Baumgardt

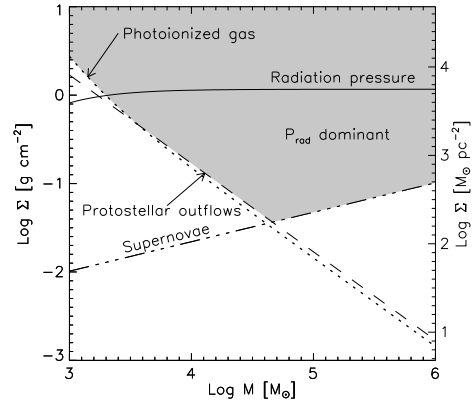


FIG. 2.— Feedback in protoclusters of mean surface density  $\Sigma$  and mass  $M$ . Radiation pressure is the dominant mechanism throughout the shaded region. The lines show where each mechanism alone achieves  $\mathcal{E} = 0.5$ . These allow for partial sampling of the stellar IMF and hence differ slightly from the power laws in Table 1 (noticeable only for  $M \lesssim 10^4 M_\odot$ ).

et al. 2008). As we have already noted, these results are expected for  $\alpha = 0$ , and they are inconsistent with the observed mass functions of young clusters.

### 3. STAR FORMATION EFFICIENCY

We now consider five specific feedback mechanisms: supernovae, main-sequence winds, protostellar outflows, photoionized gas, and radiation pressure. For the first four, we review results from the literature. Supernova feedback begins only after the  $> 3.6$  Myr lifetimes of massive stars. Unless turbulence within a protocluster is maintained by feedback or external forcing, stars would form rapidly and consume its ISM, with  $\mathcal{E} \rightarrow 1$  in  $1 - 2$  crossing times. This implies that supernovae can dominate only for  $2\tau_c \gtrsim 3.6$  Myr unless another mechanism somehow keeps  $\mathcal{E}$  small without expelling much ISM (Krumholz & Matzner 2009). However, even in this contrived situation, supernovae would play only a secondary role. Main-sequence winds are not effective if their energy is able to leak out of the bubbles they blow (Harper-Clark & Murray 2009). As a result of this leakage, winds simply provide an order-unity enhancement to radiation pressure (Krumholz & Matzner 2009). Protostellar outflows can only remove the ISM from protoclusters with escape velocities below about  $7 \text{ km s}^{-1}$  (Matzner & McKee 2000). Photoionized gas is important as a feedback mechanism only when its pressure exceeds that of radiation throughout most of an H II region. This in turn requires that the H II region be larger than the radius  $r_{\text{ch}}$  at which  $P_{\text{rad}} = P_{\text{gas}}$ , a condition harder to satisfy in massive, compact protoclusters (Krumholz & Matzner 2009).

We summarize these results in Table 1 and Figure 2. As the plot shows, the mechanisms discussed thus far are relatively ineffective in protoclusters with  $M \gtrsim 10^4 M_\odot$  and  $\Sigma \gtrsim 0.1 \text{ g cm}^{-2}$ . We therefore turn to radiation pressure. This would be an energy-driven feedback mechanism if all photons, even those re-radiated by dust grains, remained trapped within a protocluster. However, this is possible only if the protocluster is so dense and smooth that the covering fraction seen from its center exceeds  $\sim 90\%$  in the infrared (Krumholz &

TABLE 1  
FEEDBACK MECHANISMS

Mechanism	Type	Limitation	Threshold <sup>†</sup>	Evaluated <sup>†</sup>
Supernovae	Energy	Too late	$\tau_c \approx 1.8 \text{ Myr}$	$\Sigma_0 \approx 0.022 M_4^{1/3}$
Main-sequence winds	Either <sup>a</sup>	Relatively weak <sup>a</sup>	Never	...
Protostellar outflows	Momentum	Confined in massive clusters <sup>b</sup>	$V_e \approx 7 \text{ km s}^{-1}$	$\Sigma_0 \approx 0.17 M_4^{-1}$
Photoionized gas	Momentum	Crushed by $P_{\text{rad}}^c$	$S_{49} \approx 21 R_h / \text{pc}$	$\Sigma_0 \approx 0.15 M_4^{-1}$
Radiation pressure	Momentum	...	Equations (6) and (7)	$\Sigma_0 \approx 1.2$

<sup>†</sup>Parameters required for  $\mathcal{E} = 0.5$ . Evaluations assume a fully-sampled stellar IMF. Notation:  $S_{49} \equiv S/10^{49} \text{ s}^{-1}$  (ionization rate),  $M_4 \equiv M/10^4 M_\odot$ ,  $\Sigma_0 \equiv \Sigma/\text{g cm}^{-2}$ .

<sup>a</sup>Stellar winds are energy-driven and dominant if trapped, but are expected to leak, making them momentum-driven and weak.

<sup>b</sup>Based on Equation (55) of Matzner & McKee (2000), updated with  $f_w v_w = 80 \text{ km s}^{-1}$  (Matzner 2007).

<sup>c</sup>Based on Equation (4) of Krumholz & Matzner (2009) for the blister case, with the coefficient reduced by a factor of  $2.2^2$  to correct an error in the published paper and updated with  $\langle L/M_* \rangle = 1140 L_\odot M_\odot^{-1}$  and  $\langle S/M_* \rangle = 6.3 \times 10^{46} \text{ s}^{-1} M_\odot^{-1}$  (Murray & Rahman 2010).

Matzner 2009). More realistically, the protocluster would be porous enough that photons could escape after only a few interactions with dust grains, and radiation pressure would then be a momentum-driven feedback mechanism. The following analysis extends that of Elmegreen (1983), Scoville et al. (2001), Thompson et al. (2005), Krumholz & Matzner (2009), and Murray et al. (2010).

We consider an idealized, spherical cloud of mass  $M$  and outer radius  $R$ , with an internal density profile  $\rho \propto r^{-k}$  (hence  $R_h = 2^{-1/(3-k)} R$ ). Radiation from young stars near the center of the cloud ionizes the gas and drives the expanding outer shell of this H II region. After a time  $t$ , the momentum imparted to the shell is  $p_s = f_{\text{trap}} L t / c$ , where  $L$  is the stellar luminosity (assumed constant for simplicity), and  $f_{\text{trap}} \sim 2 - 5$  accounts for assistance from main-sequence winds and incomplete leakage of starlight and wind energy (Krumholz & Matzner 2009). Neglecting gravity for the moment, the velocity and radius of the shell are related by  $v_s = \eta r_s / t$  with  $\eta = 2/(4 - k)$ . Thus, when the shell reaches the cloud surface ( $r_s = R$ ), it has swept up all the remaining ISM, with mass  $M_g = (1 - \mathcal{E})M$ , and has a velocity given by

$$v_s^2(R) = \frac{\eta f_{\text{trap}} L R}{c(1 - \mathcal{E})M}. \quad (4)$$

We specify the condition for ISM removal against gravity of the protocluster by  $v_s^2(R) = \alpha_{\text{crit}} G M / (5R)$ , where  $\alpha_{\text{crit}}$  is a parameter of order unity that accounts for magnetic support and other uncertain factors (discussed below). The required luminosity, from equation (4), is

$$L = \frac{\alpha_{\text{crit}} G c (1 - \mathcal{E}) M^2}{5 \eta f_{\text{trap}} R^2}. \quad (5)$$

The fundamental scaling  $L \propto (M/R)^2 \propto V_m^4$  arises here in the same way it does for the growth of supermassive black holes and galactic spheroids (Fabian 1999; King 2003; Murray et al. 2005). Rewriting Equation (5) in terms of  $\Sigma = M/(\pi R^2)$  and  $M_* = \mathcal{E}M$  and solving for  $\mathcal{E}$ , we obtain our basic result

$$\mathcal{E} = \frac{\Sigma}{\Sigma + \Sigma_{\text{crit}}}, \quad (6)$$

with

$$\Sigma_{\text{crit}} = \frac{5 \eta f_{\text{trap}} (L/M_*)}{\pi \alpha_{\text{crit}} G c} \approx 1.2 \left( \frac{f_{\text{trap}}}{\alpha_{\text{crit}}} \right) \text{g cm}^{-2}. \quad (7)$$

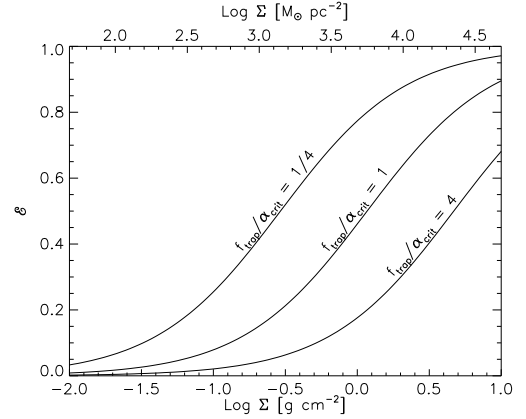


FIG. 3.— Star formation efficiency  $\mathcal{E}$  as a function of mean surface density  $\Sigma$ , computed from Equations (6) and (7) with the indicated values of  $f_{\text{trap}}/\alpha_{\text{crit}}$ .

The coefficient in the last equation is based on  $\eta = 2/3$  and  $L/M_* = 1140 L_\odot / M_\odot$  (see notes to Table 1). Regardless of the exact value of  $f_{\text{trap}}/\alpha_{\text{crit}}$ , we note that  $\mathcal{E}$  depends on  $M$  and  $R$  only through  $\Sigma$ . Thus, when  $\Sigma$  is constant,  $\mathcal{E}$  is independent of  $M$ , and the mass functions of clusters and clouds have the same exponent ( $\beta_* = \beta \approx \beta_o$ ).

Figure 3 shows  $\mathcal{E}(\Sigma)$  computed from Equations (6) and (7). Clearly,  $\mathcal{E}$  increases monotonically with  $\Sigma$  from 0 to 1, reaching  $\mathcal{E} = 0.3$  for  $\Sigma \sim 0.5 (f_{\text{trap}}/\alpha_{\text{crit}}) \text{ g cm}^{-2}$ . We expect  $f_{\text{trap}} \sim \alpha_{\text{crit}} \sim 2 - 5$ . The escape velocity from the surface of an unmagnetized cloud corresponds to  $\alpha_{\text{crit}} = 10$ , while the internal velocity dispersion, possibly sufficient for some ISM removal, corresponds to  $\alpha_{\text{crit}} \approx 1.3$ . A shell driven by a constant force requires  $\alpha_{\text{crit}} = 2.3$  (for  $k = 1$ ; see Equation (A17) of Matzner & McKee (2000)). We consider  $\alpha_{\text{crit}} \approx 2$  to be plausible; certainly a protocluster boils violently and loses mass rapidly using this condition. Our intent here is not to make a detailed comparison between the model and observations. Given the simplicity of the former and the uncertainties in the latter, it is gratifying that they agree even roughly with each other.

#### 4. CONCLUSIONS

This Letter contains two main results. The first is the relation between the power-law exponents of the mass functions of molecular clouds and young star clusters,  $\beta_o$  and  $\beta_*$ , in the limiting regimes in which stellar feedback is energy-driven and momentum-driven, Equations (3a) and (3b), which bracket all realistic cases. The predicted  $\beta_*$  depends significantly on the initial size-mass relation of the protoclusters. We find good agreement between the predicted and observed  $\beta_*$ , especially for momentum-driven feedback, for  $\Sigma \propto M/R_h^2 \approx \text{constant}$ , the relation indicated by observations of gas-dominated protoclusters. In this case, the star formation efficiency is independent of protocluster mass, ensuring that the fraction of clusters that remain gravitationally bound following ISM removal is also independent of mass.

The second main result is an estimate of the star formation efficiency in protoclusters regulated by radiation

pressure, Equations (6) and (7). This is likely to be the dominant feedback process in massive protoclusters. We show that  $\mathcal{E}$  depends on  $M$  and  $R_h$  only through the mean surface density  $\Sigma$ , which in turn guarantees consistency between the observed power-law exponents of the mass functions of molecular clouds and young star clusters according to our general relations. For  $\Sigma \sim 1 \text{ g cm}^{-2}$ , we estimate  $\mathcal{E} \sim 0.3$ , in satisfactory agreement with observations.

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